Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Metric Space

Subject Code : 4SC05MSC1 **Branch: B.Sc. (Mathematics)**

Semester : 5 **Date : 27/03/2018** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

1. (1) Use of Programmable calculator & any other electronic instrument is prohibited.
2. (2) Instructions written on main answer book are strictly to be obeyed.
3. (3) Draw neat diagrams and figures (if necessary) at right places.
4. (4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Discrete metric space. (01)
b) What is IntA ? where A=(-1,2] . (01)
c) True/false: Every open interval in real line R is open set. (01)
d) What is derived set? (01)
e) Define: Open set. (01)
f) What is closer of Closed set? (01)
g) True/false :Every closed sphere is an closed set (01)
h) What is open sphere in R^2. (01)
i) Define : Equivalent metrics. (01)
j) What is union of any set with it's derived set. (01)
k) Find $\bar{A}$ if A=[0,1)∩Q (01)
l) Define : Closed set . (01)
m) True/false: Closer of any set is always closed. (01)
n) If A is any closed set in R then what will be extA? (01)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

(a) What is metric space? The function d defined by **(07)**
d(x,y)=$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for all x,y ∈ R^2,Where x =(x_1 , y_1) and
y ==(x_2 , y_2) Show that (R^2,d) is metric space.

(b) Show that C[a,b] with metric d(f,g)=sup$|f(u) - g(u)|$ for all u in [a,b] **(07)**

Q-3 **Attempt all questions** **(14)**

(a) What is closed set? **(07)**
Which of the following sets are closed sets.
(i) $(-2,2) \cap Q$ on R,
(ii) { (x, y) / x = y } on R^2,
(iii) { (x, y) / $x^2 + y^2 \geq \sqrt{2}$ } on R^2.

(b) Let the set l_∞ of all bounded sequences $\{x_n\}$ of real number with the function d defined by d($\{x_n\}$, $\{y_n\}$) = sup{$|x_n - y_n|$, n ∈ N } , show that (l_∞ , d) is metric space. **(07)**

Q-4 **Attempt all questions** **(14)**

(a) If d_1 and d_2 are two metric space on X , then show that (X, $md_1 + n\, d_2$) is metric space ,where m , n ϵ N but (X , ${d_1}^2$) is not a metric space. **(07)**

(b) Which of the following sets are open sets? Explain with figure. **(07)**

(i) R~ $[-2,1]$ on R,

(ii) { (x, y) / $x = y$ } on R^2.

(iii) R^2~ { (x, y) / $x - y = 1$ } on R^2.

(iv) { (x, y) / $x + y > 1$ } on R^2.

Q-5 **Attempt all questions** **(14)**

(a) What is accumulation point of the set ? find A' for the following sets **(07)**

(1) (R , d_u) , A=[0,1]∪{2,3,4}

(2) (R , d_u) , A=(0,1]

(3) (R , d_u) , A={1,$\frac{2}{3}$,$\frac{3}{4}$,$\frac{4}{5}$,……..}.

(b) Let X, d be a metric space and $Y \subseteq X$, then prove that a subset to be open in (Y, d_Y) if and only if there exists a set G open in (X, d) such that $A = G \cap Y$. **(07)**

Q-6 **Attempt all questions** **(14)**

(a) Let A be a subset of a metric space X. Prove that A is closed if and only if it contains ∂A. **(07)**

(b) What is exterior point of subset A ? Explaine intA ,extA , bdA for the following. **(07)**

(1) X=R^2 , A= open annulus having centre at origin and radii are 2 and 5

(2) X=R^3 , A= Closed sphere centre at pole with radius 2.

Q-7 **Attempt all questions** **(14)**

(a) State and prove Heine-borel theorem **(07)**

(b) What is continuous function in metric space ? Show that the function f:(R,d_1) → (R, d_2); f(x) = x is continuous ; Where d_1= d_2= usual metric . **(04)**

(c) Show that image of Cauchy sequence under continuous function need not be Cauchy. **(03)**

Q-8 **Attempt all questions** **(14)**

(a) The continuous image of a compact set is compact. **(06)**

(b) Let (X, d), (Y, d') be metric spaces and f : X → Y be a map. Then show that f is continuous if for every convergent sequence (x_n) in X lim f(x_n) = f(lim x_n) . **(06)**

(c) What do you mean by Pseudo metric space ? **(02)**